# USING SPATIAL SYMMETRIES TO DO RESOURCE ALLOCATION IN LARGE SCALE SYSTEMS

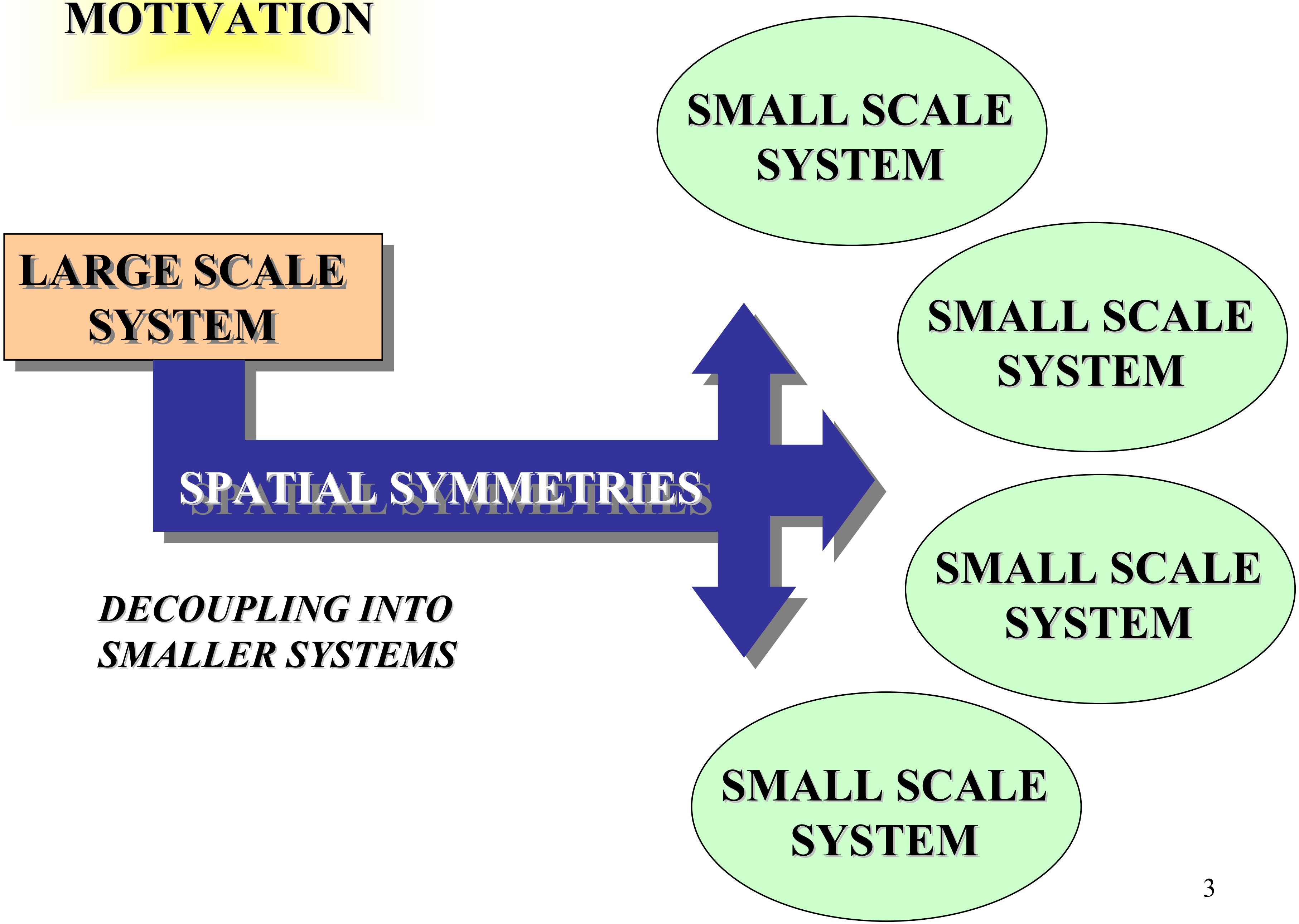
# OBJECTIVES

*SOLVE THE RESOURCE ALLOCATION PROBLEM FOR LARGE SCALE LINEAR SYSTEMS. (CONTROL WITH FLOW CONSTRAINTS)*

*USE SYSTEM SPATIAL SYMMETRIES TO "BREAK" CONTROLLER DESIGN AND IMPLEMENTATION INTO SMALLER PROBLEMS*

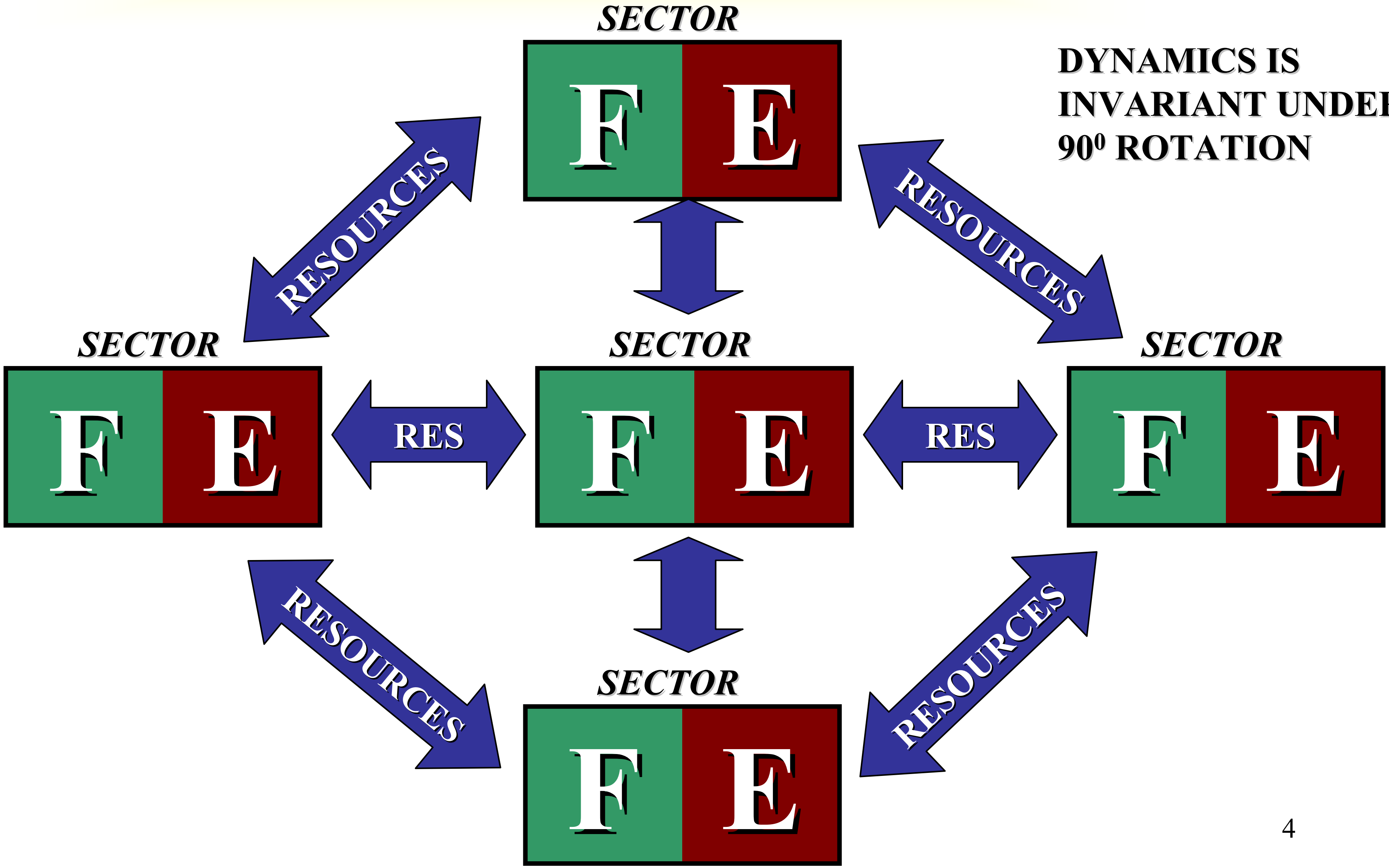
*USE SYSTEM SPATIAL SYMMETRIES AGAIN TO ISOLATE FLOW CONSTRAINTS*

# MOTIVATION



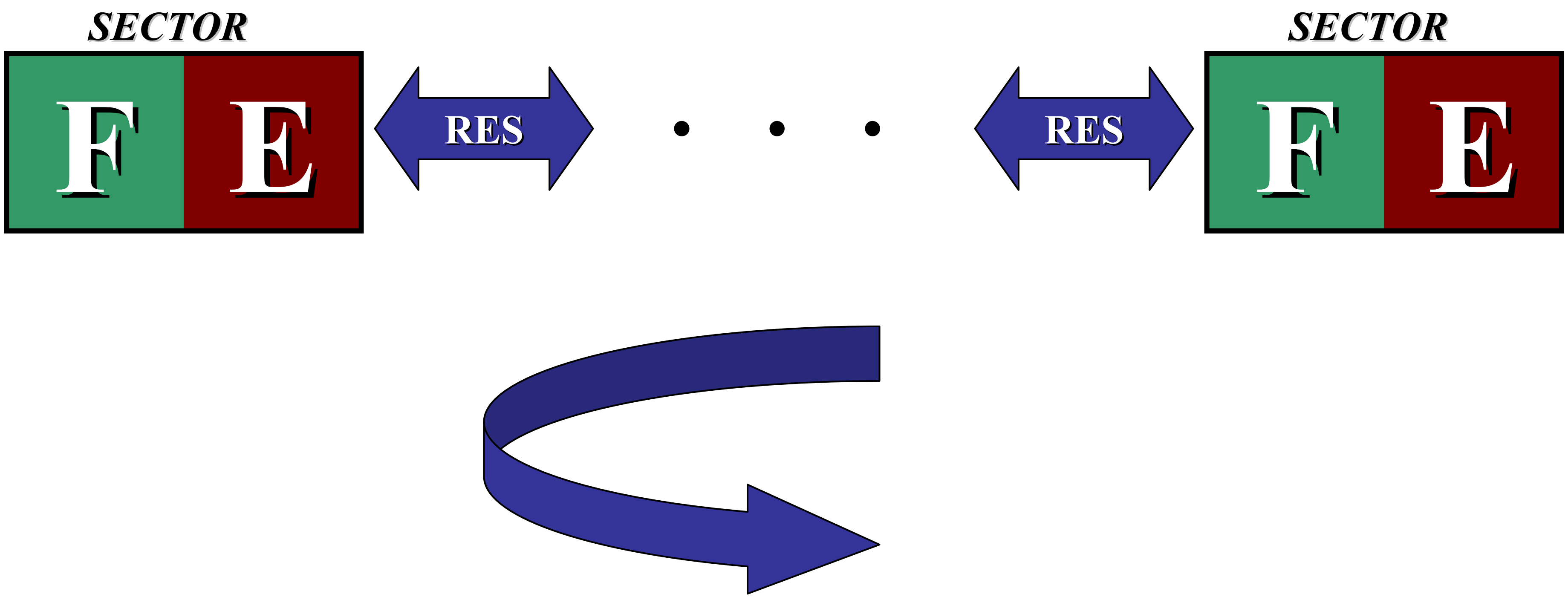
SMALL SCALE SYSTEM
LARGE SCALE SYSTEM
SMALL SCALE SYSTEM
SPATIAL SYMMETRIES
SMALL SCALE SYSTEM
DECOUPLING INTO SMALLER SYSTEMS
SMALL SCALE SYSTEM

# EXAMPLES OF SYSTEMS WITH SYMMETRY




**DYNAMICS IS INVARIANT UNDER $90^0$ ROTATION**

# EXAMPLES OF SYSTEMS WITH SYMMETRY




***INVARIANT UNDER REFLECTION AND PERMUTATIONS***

# EXAMPLES OF SYSTEMS WITH SYMMETRY

***EACH SMALL SCALE SYSTEM GOVERNS THE DYNAMICS OF THE LARGE SCALE SYSTEM OVER A SPATIAL EIGENVECTOR.***

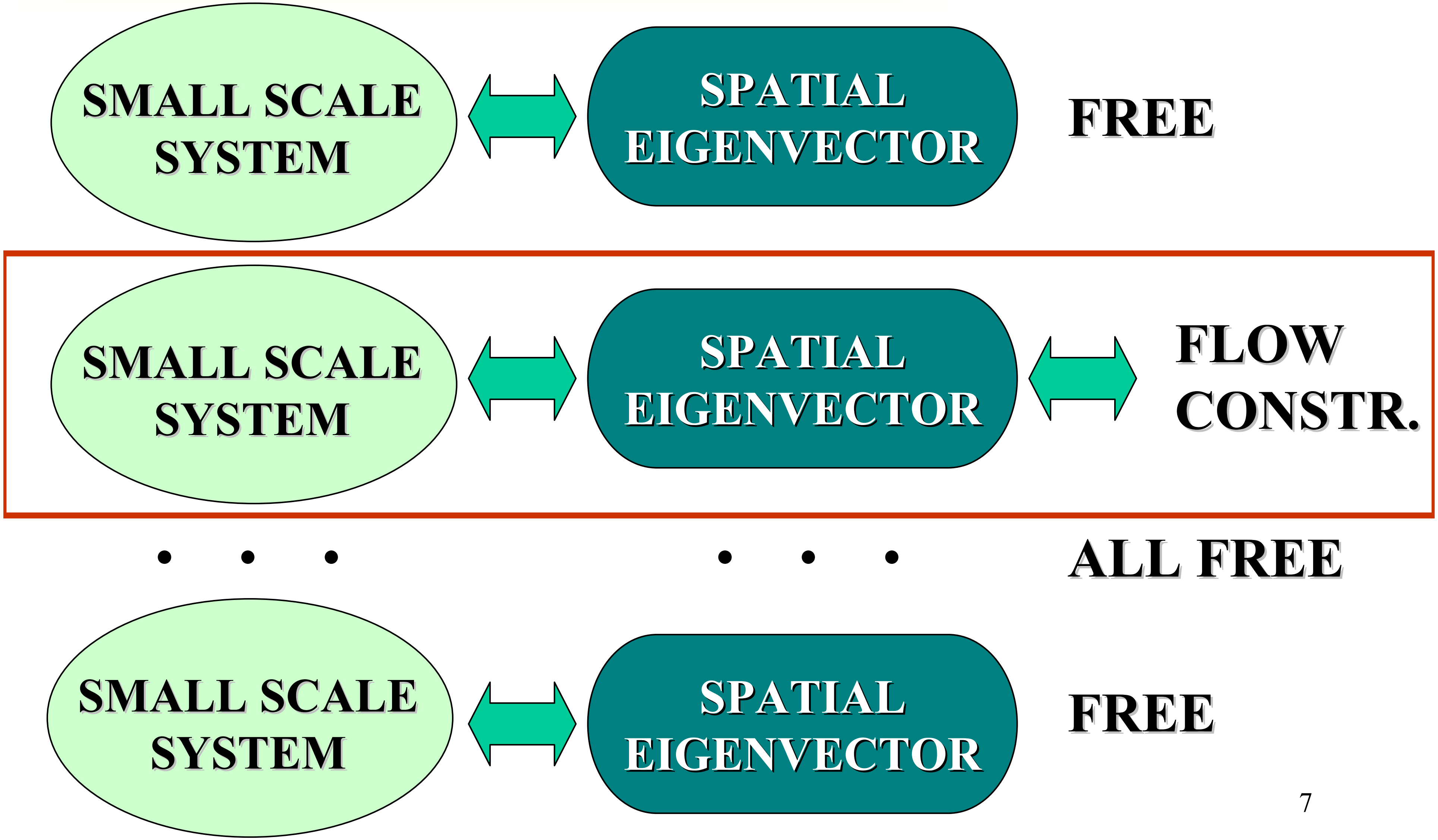
***THE COMBINED EFFECT OF ALL SMALL SCALE SYSTEMS DESCRIBES THE DYNAMICS OF THE OVERALL SYSTEM.***

***EXAMPLES OF SPATIAL EIGENVECTORS ARE:***

**1 1 1 … 1 1 1** → ***PHYSICAL MEANING: SPATIAL MEAN VALUE OVER ALL SECTORS.***

**1 -1 0 .. 0 0 0** → ***PHYSICAL MEANING: SHIFT OF RESOURCES FROM SECTOR 1 TO 2.***

# ISOLATING FLOW CONSTRAINTS



# ISOLATING FLOW CONSTRAINTS

***LINEAR FLOW CONSTRAINTS CAN BE EASILY DEALT WITH , WHENEVER THEY CAN BE ISOLATED IN ONE OF THE SPATIAL EIGENVECTORS***

***AS AN EXAMPLE, THE FOLLOWING FLOW CONSTRAINT:***

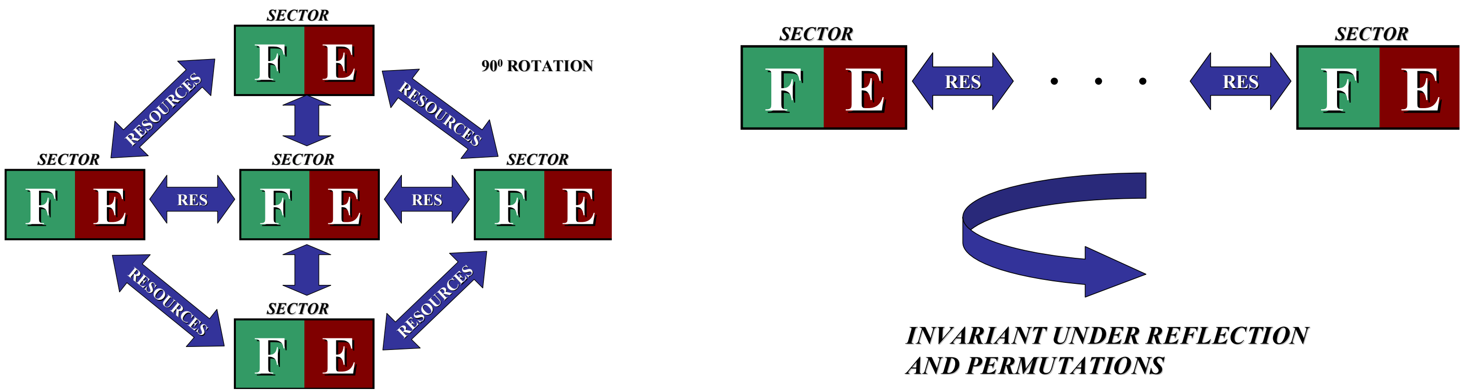
$$\sum_i flow_{\text{sector } i} = \text{constant}$$

***COMPLETELY CONSTRAINS THE BEHAVIOUR OF THE SYSTEM ALONG THE FOLLOWING SPATIAL EIGENVECTOR :***

**[1 1 1 … 1 1 1]**

***THE REMAINING SPATIAL EIGENVECTORS ARE NOT INFLUENCED BY THE CONSTRAINT, PROVIDED THAT THEY ORTHOGONAL TO [1…1].***

# ISOLATING FLOW CONSTRAINTS




**BOTH EXAMPLES ADMITT A SPATIAL EIGENVECTOR ASSOCIATED WITH THE TOTAL FLOW CONSTRAINT:**

$$\sum_i flow_{\text{sector } i} = \text{constant}$$

# CONTROLLER DESIGN PROCEDURE

COMPUTE SPATIAL EIGENVECTORS

CHOOSE PHYSICALLY MEANINGFULL DECOUPLING EIGENVECTORS

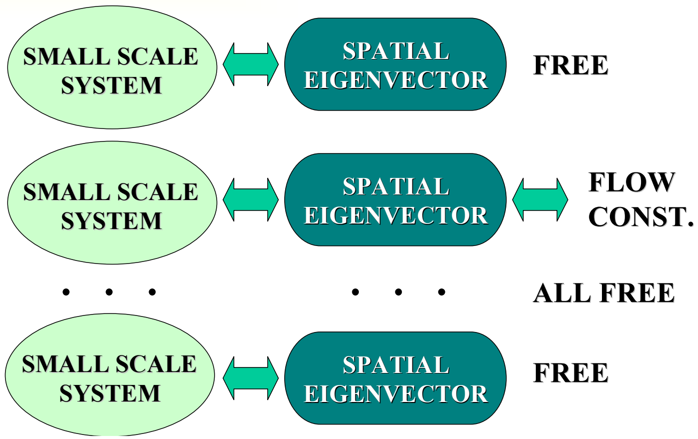
!

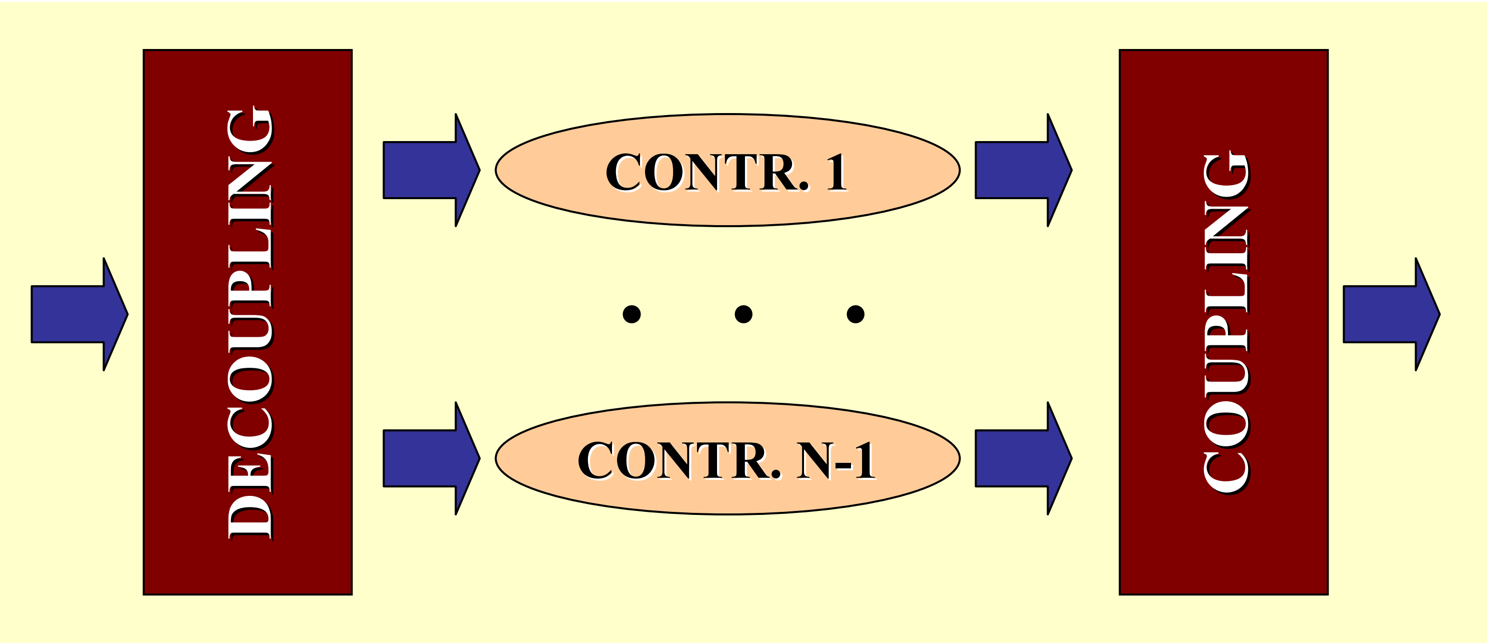
*ONE OF THE EIGENVECTORS MUST BE ASSOCIATED WITH THE FLOW CONST.*

DESIGN INDEPENDENTLY ONE CONTROLLER FOR EACH FREE SPATIAL EIGENVECTOR (NOT ASSOCIATED WITH FLOW CONSTR.)

# CONTROLLER IMPLEMENTATION

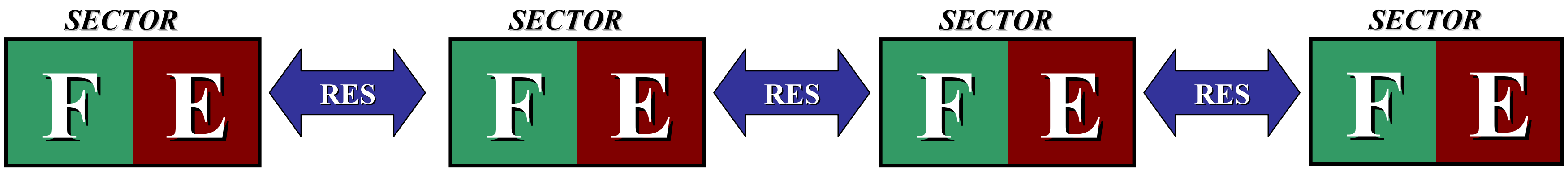
***ASSUME N SMALL SCALE SYSTEMS AND ONE FLOW CONSTRAINT.***
***THIS LEADS TO N-1 INDEPENDENT CONTROLLERS.***



# SIMULATION EXAMPLE

$$\sum_i flow_{\text{sector } i} = \text{constant}$$




***4 SPATIAL EIGENVECTORS:***

***3 ACCOUNT FOR THE RESOURCE TRANSFER BETWEEN NEIGHBORING SECTORS***

***1 ACCOUNT FOR THE FLOW CONSTRAINT:***

# SIMULATION EXAMPLE: SECTOR MODEL

MAIN RESOURCE.(OUTPUT)

SECONDARY RESOURCE.

FRIENDLY

$$\dot{x}_1^f = -a_1^f x_1^f + a_2^f x_2^f - a_e^f x_1^e$$

CONSUMPTION. ENEMY EFFECT.

$$\dot{x}_2^f = -b_2^f x_1^f - b_e^f x_e^f + u$$

CONTROL (ALLOCATION DECISION)

ENEMY

$$\dot{x}_1^e = -a_1^e x_1^e + a_2^e x_2^e - a_f^e x_1^f$$

UNKNOWN DISTURB.

$$\dot{x}_2^e = -b_2^e x_1^e - b_f^e x_f^e + d$$

# SIMULATION EXAMPLE: SETUP

USING 3 H-INF CONTROLLERS TO MINIMIZE THE "AMPLIFICATION" FROM THE DISTURBANCE d TO A LINEAR FUNCTION OF THE MAIN RESOURCES.

**3 DIFFERENT OBJECTIVES WERE SIMULATED:**

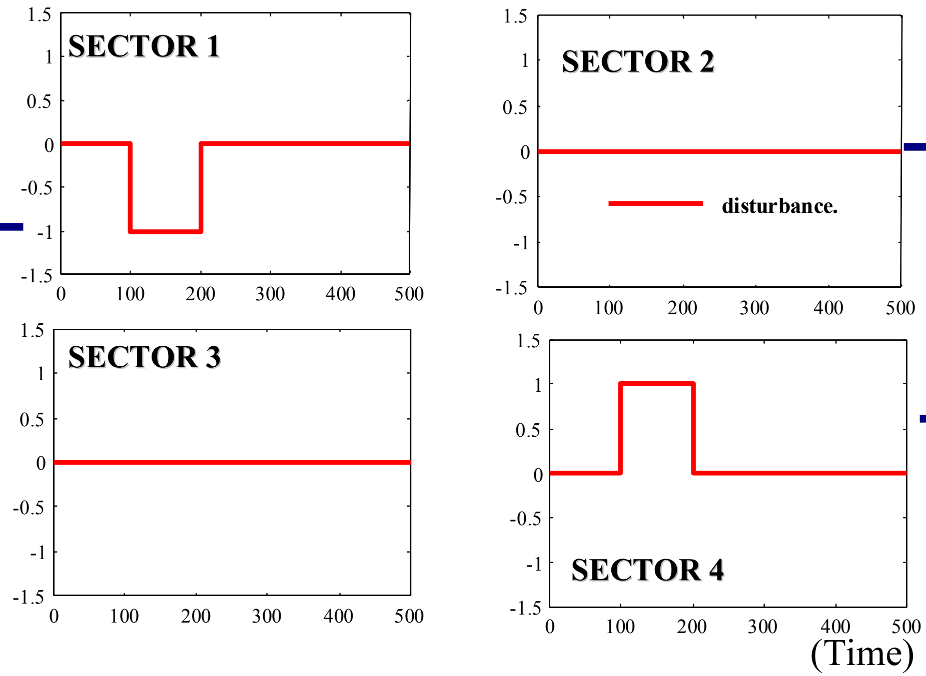
- ENGAGING THE ENEMY
- NEUTRALIZE ENEMY PLAN
- REDUCE INFLUENCE OF ENEMY STRATEGIES

# SIMULATION EXAMPLE: DISTURBANCE

***IN THE FOLLOWING SIMULATION SETUP, THE ENEMIE'S ALLOCATION IS REGARDED AS AN UNKNOWN DISTURBANCE.***

***THE SUBSEQUENT RESULTS WERE OBTAINED ASSUMING THE ENEMY DECIDED TO TRANSFER RESOURCES FROM SECTOR 1 TO 4.***

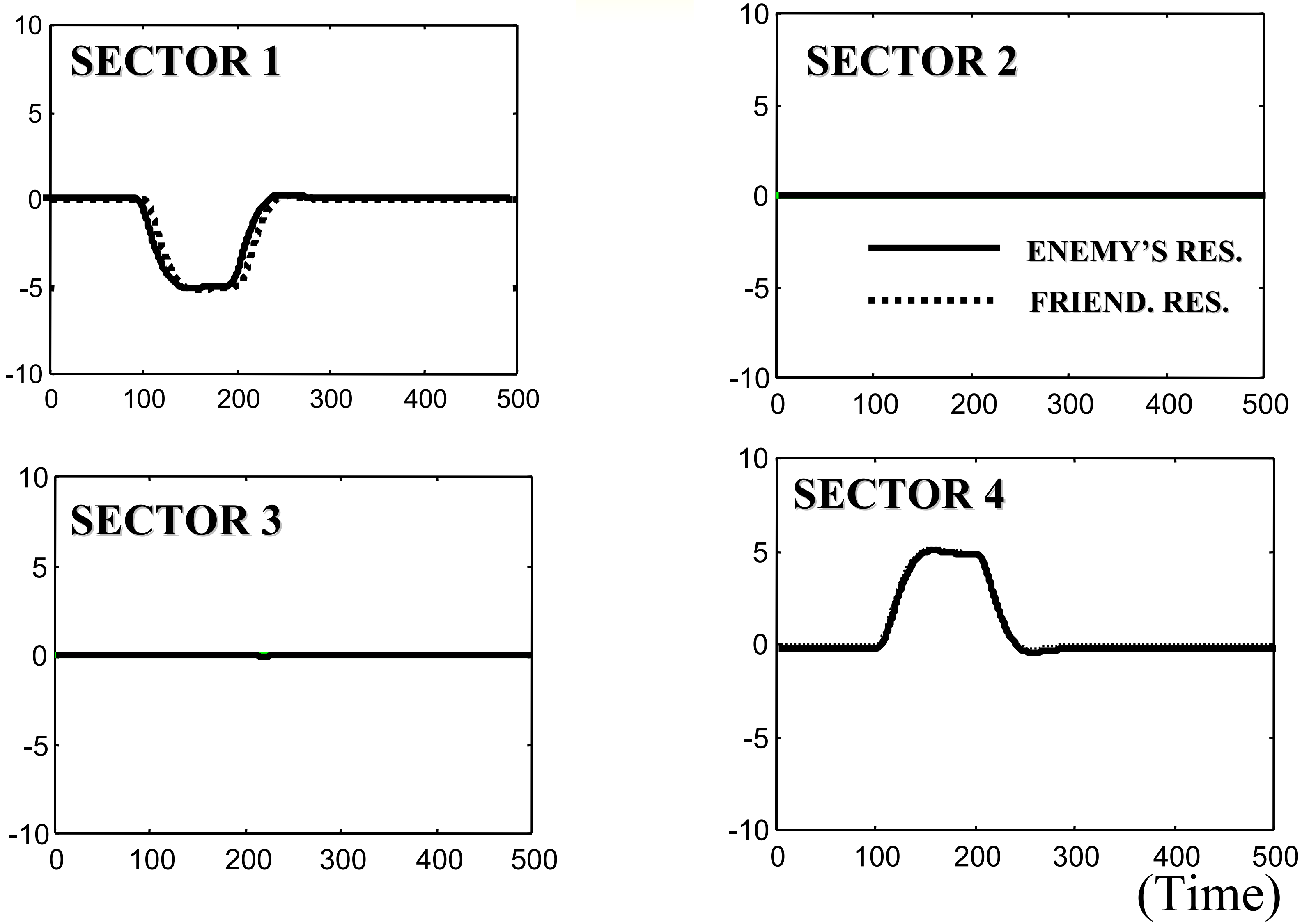
***RESOURCES LEAVING SECTOR 1***



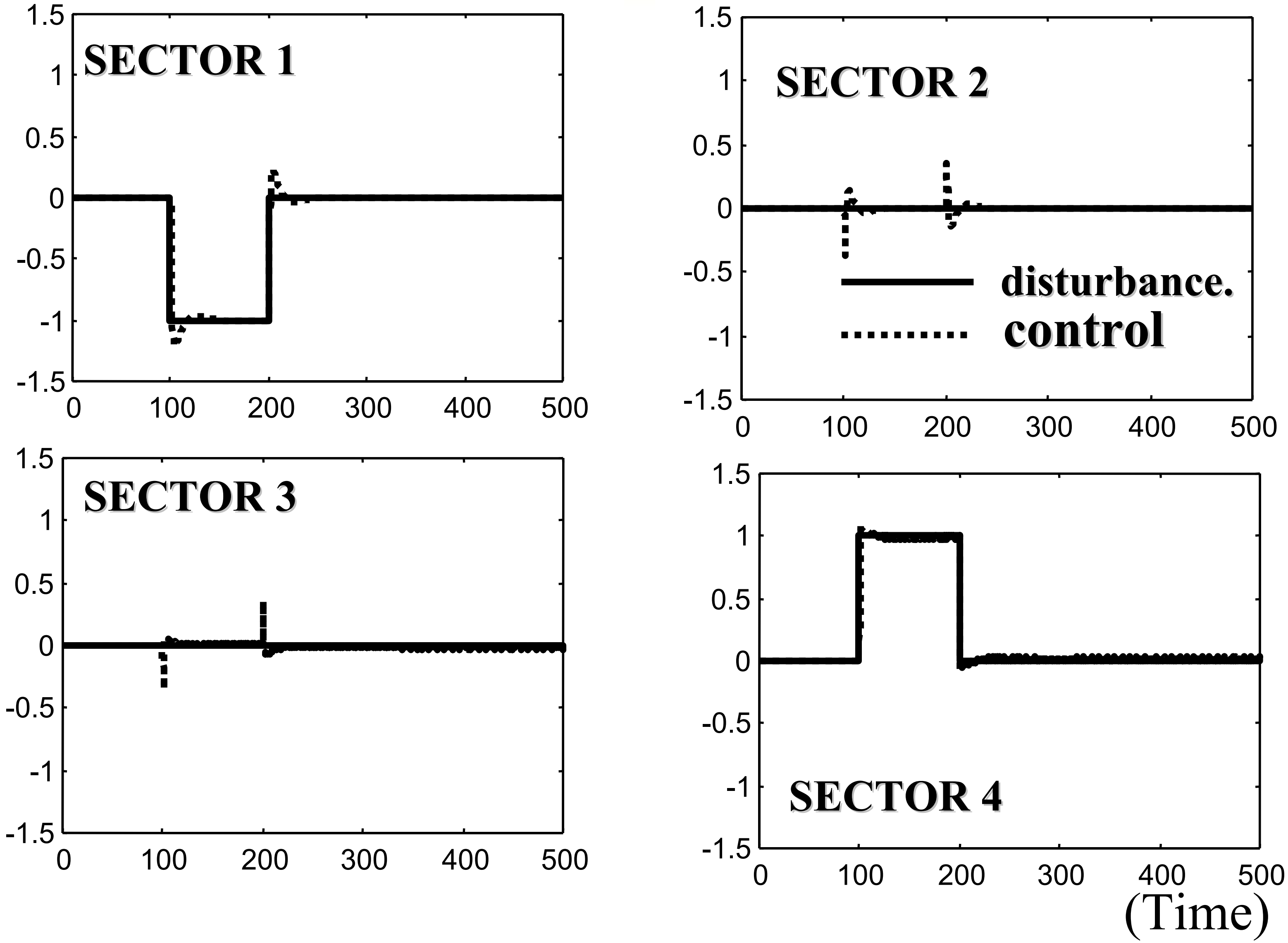

***ZERO REPRESENTS NOMINAL***

***RESOURCES ARRIVING AT SECTOR 4***

# SIMULATION EXAMPLE: RESULTS - ENGAGING THE ENEMY



# SIMULATION EXAMPLE: RESULTS - ENGAGING THE ENEMY



## SIMULATION EXAMPLE: RESULTS - ENGAGING THE ENEMY

***COMMENTS:***

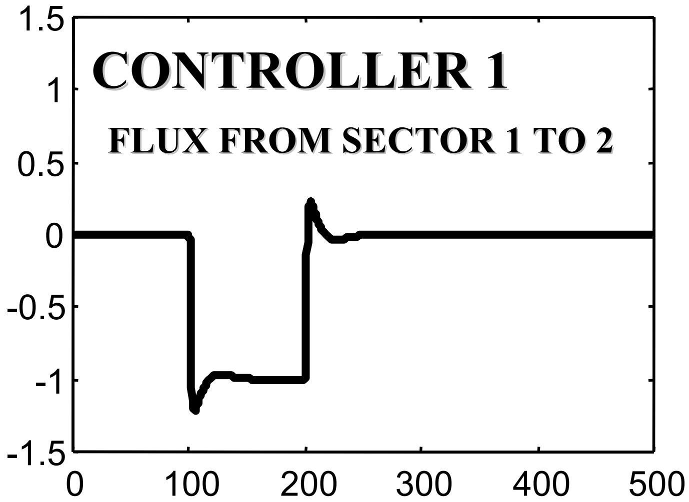
***BY INSPECTION IT IS POSSIBLE TO INFER:***

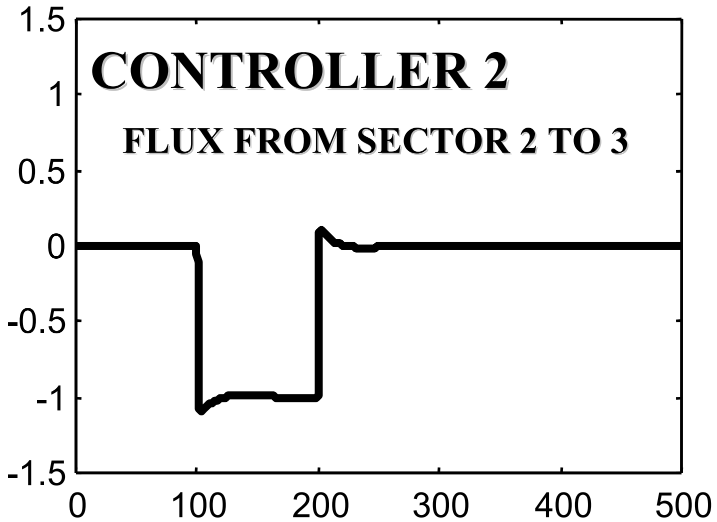
- ***(SLIDE 16) THAT THE FRIENDLY'S MAIN RESOURCE TRACKS CLOSELY THE ENEMY'S MAIN RESOURCE.***

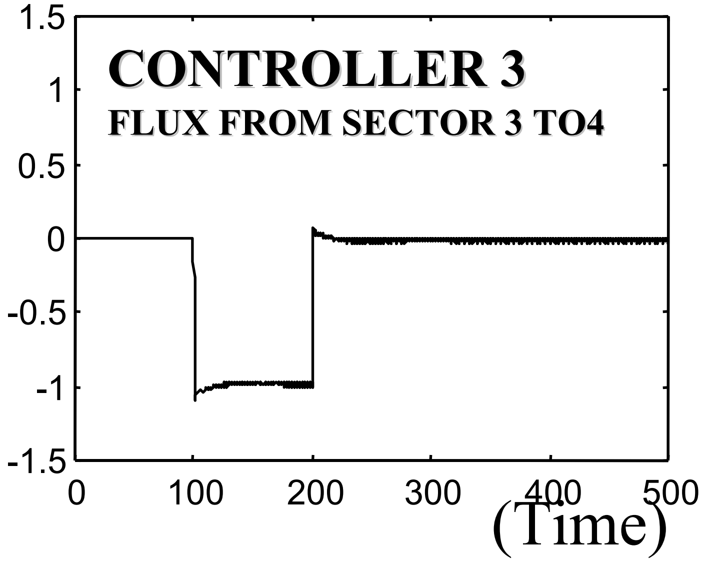
- ***(SLIDE 17) THAT THE FRIENDLY'S ALLOCATION IS VERY SIMILAR TO THE ENEMY'S ALLOCATION. THIS SHOWS THAT FROM THE MEASUREMENTS OF MAIN RESOURCES, THE CONTROLLER IS ABLE TO FIGURE OUT WHAT WAS THE ENEMY'S STRATEGY AND ACT ACCORDINGLY.***

## SIMULATION EXAMPLE: RESULTS - ENGAGING THE ENEMY

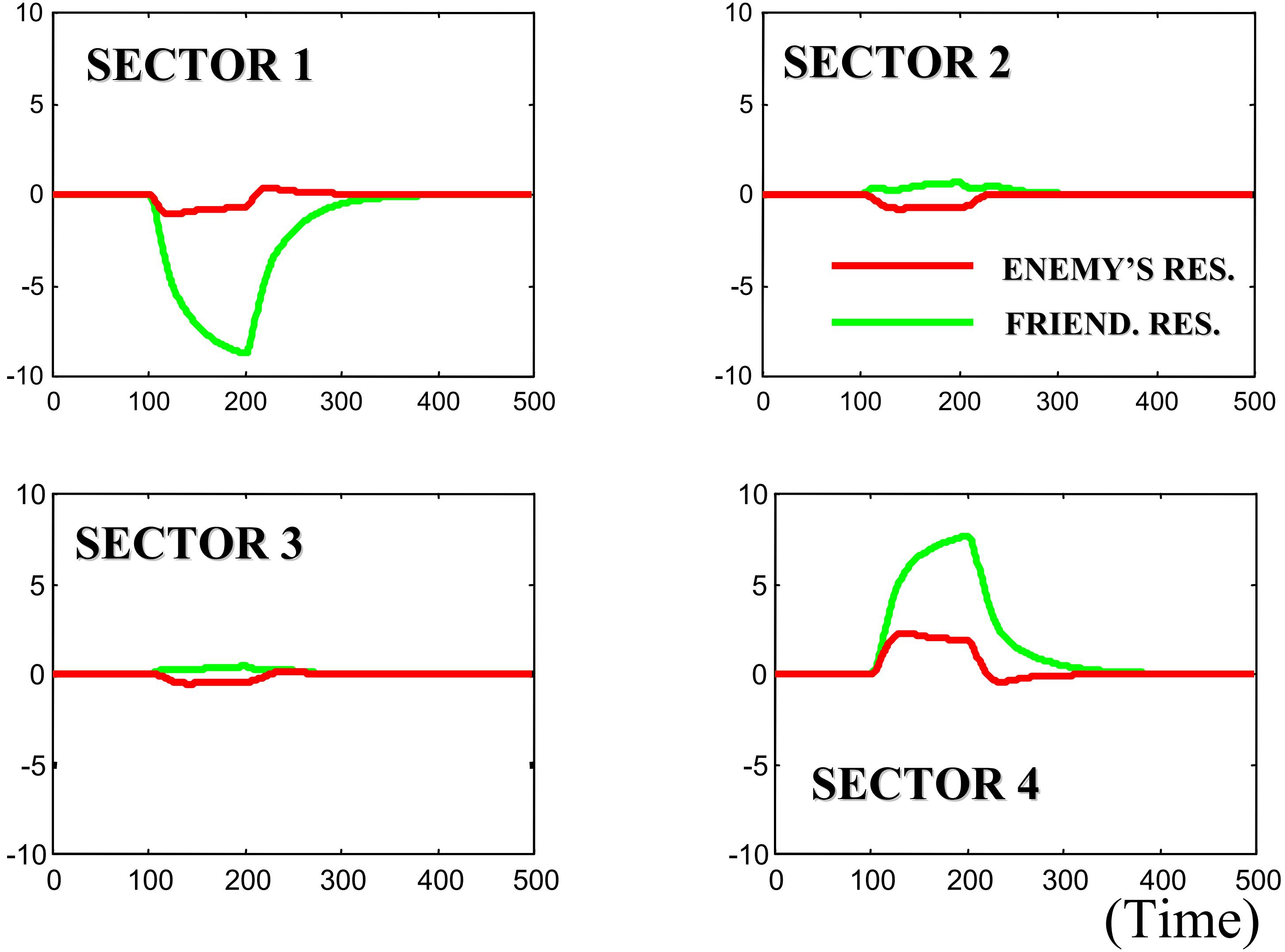
**DECOUPLED CONTROLLERS**



# SIMULATION EXAMPLE: RESULTS - ENEMY REDUCTION



# SIMULATION EXAMPLE: RESULTS - FRIEND. REDUCTION